Warm-Up

CST/CAHSEE:

What type of triangle is formed by the points $A(4,2)$, $B(6,-1)$, and $C(-1, 3)$?

A. right

B. equilateral

C. isosceles

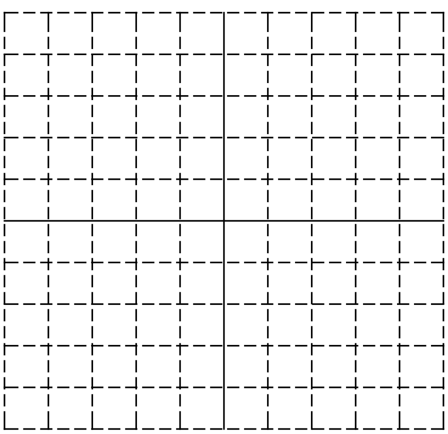
D. scalene

Review: Grade

Find the distance between the points $(-2, 5)$ and $(3, -1)$

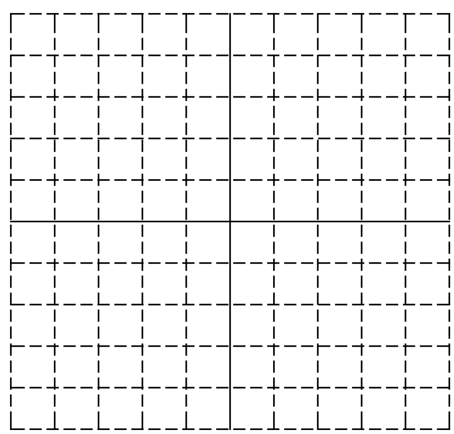
Current: Grade

Graph $\frac{1}{4}y = x^2$



Other: Grade

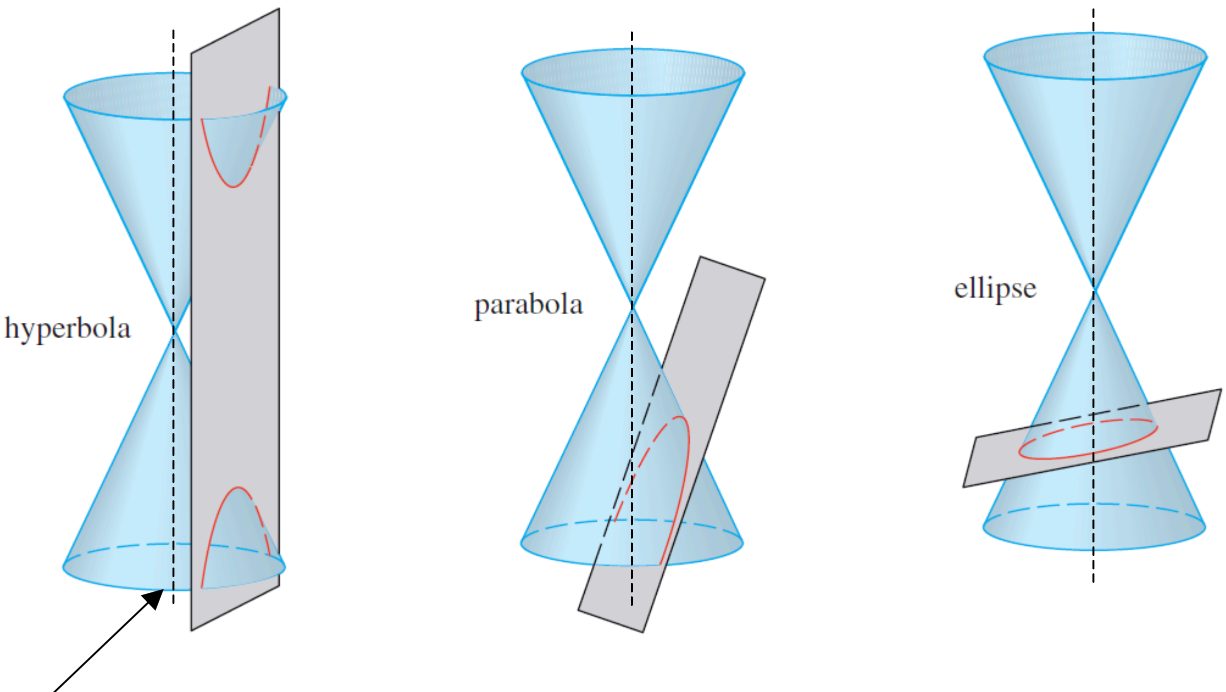
a) Graph $x = y^2$



b) What is the difference between the graph in quadrant 3 and the graph in quadrant 4?

Introduction to Conics

In general, a conical surface consists of two congruent unbounded halves joined by the apex, or vertex. Each half is called a nappe.



The axis of the cone is a vertical line that passes through the vertex.

Conic sections are formed by cutting the cone with a plane.

Hyperbola: formed by cutting the cone with a plane parallel to the axis.

Parabola: formed by cutting the cone with a plane parallel to the edge of the cone.

Ellipse: formed by cutting the cone with an inclined plane, neither parallel to the axis nor the edge. (A circle is a special case of an ellipse, where the inclined plane is parallel to the base of the cone.)

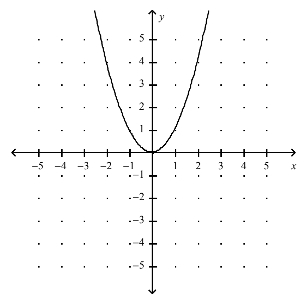
Parabolas

Think-Pair-Share:

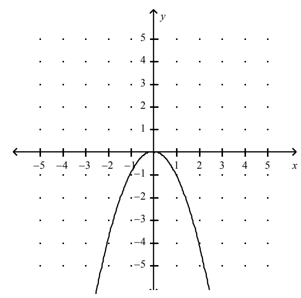
Tell all you know about the graph of $y = ax^2$

Have students discuss with a partner, then share with the class. Look for these specific ideas

- a parabola
- concave up if $a > 0$ or down if $a < 0$
- vertex (0,0)
- axis of symmetry at $x = 0$



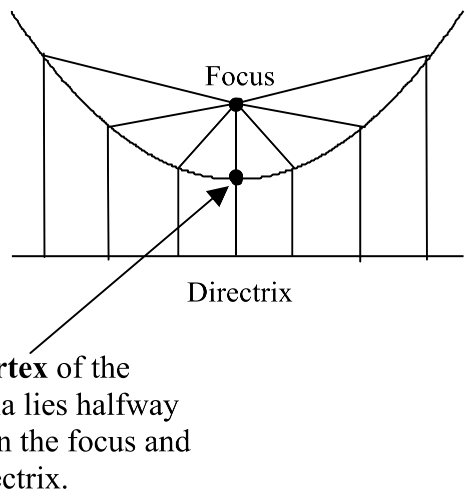
$a > 0$



$a < 0$

We will now define a parabola:

Definition : A **<u>parabola</u>** is a locus* of points in a plane that are equidistant from a point called the **focus** and a line called the **directrix.**



Use the Concentric Circles Activity to reinforce the definition of a parabola.

*Locus: a collection of points which share one or more properties

Concentric Circles Activity: Sketching a Parabola

Each diagram below shows several concentric circles centered at point *F*. The radii of the circles increase by 1 unit. The horizontal lines are also spaced 1 unit apart, with each line (except the one passing through *F*) tangent to a circle.

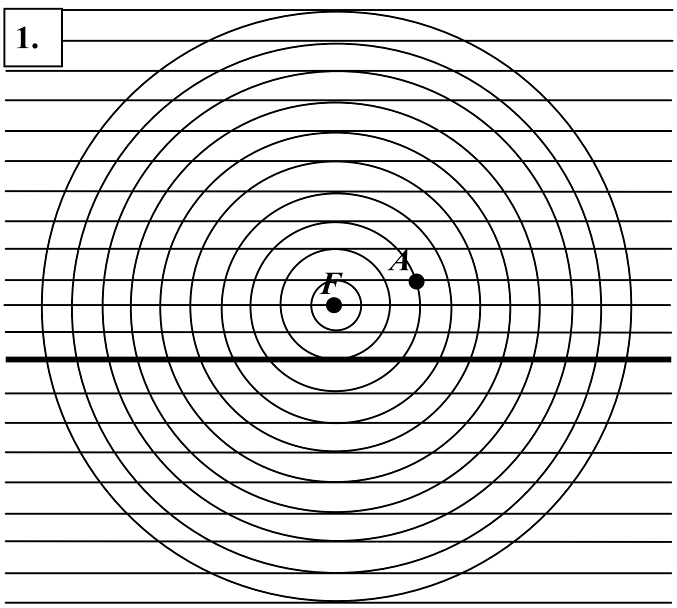
On each diagram, use the <u>definition of a parabola</u> to sketch a parabola. Use the bold line as the directrix, and point *F* as the focus for each parabola.

Procedure:

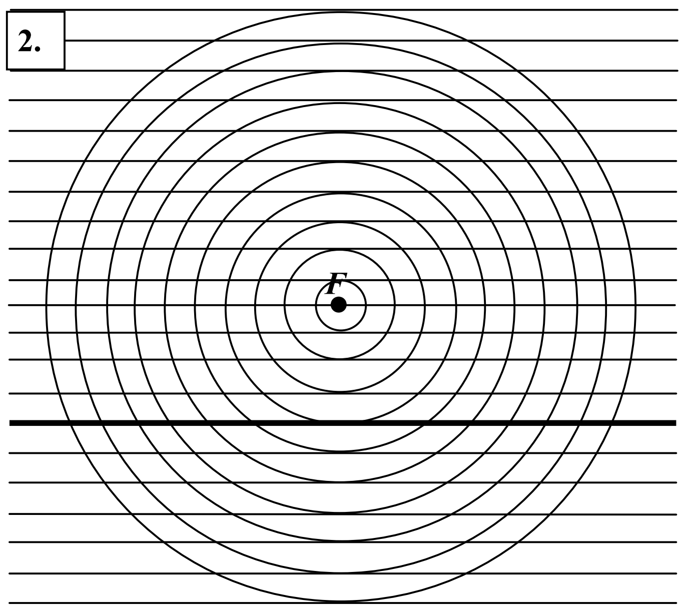
1) Discuss with a partner: On diagram **1**, how many units apart are points *F* and *A*? How many units apart are point *A* and the bold line? How does this correspond to the definition of a parabola?
2) Locate and mark at least 15 points (including the vertex) that sit on a parabola with focal point *F*and the bold line as its directrix. Discuss with a partner how you are finding those points.
3) Using the points as you found as a guide, sketch the parabola.

Repeat the process for the other diagrams, using the bold line as the directrix, and point *F* as the focus for each parabola.

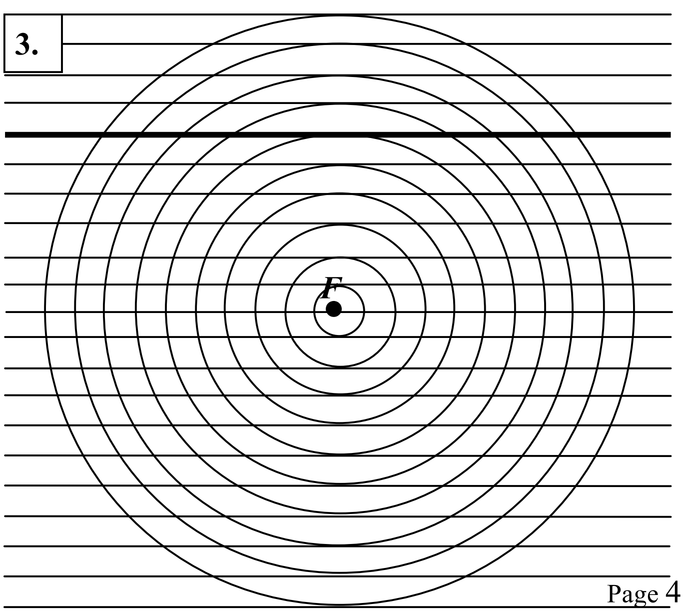
1.



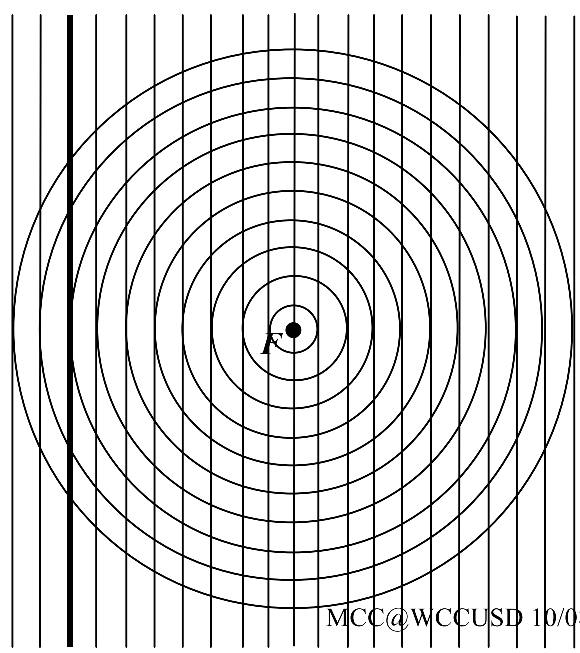
2.



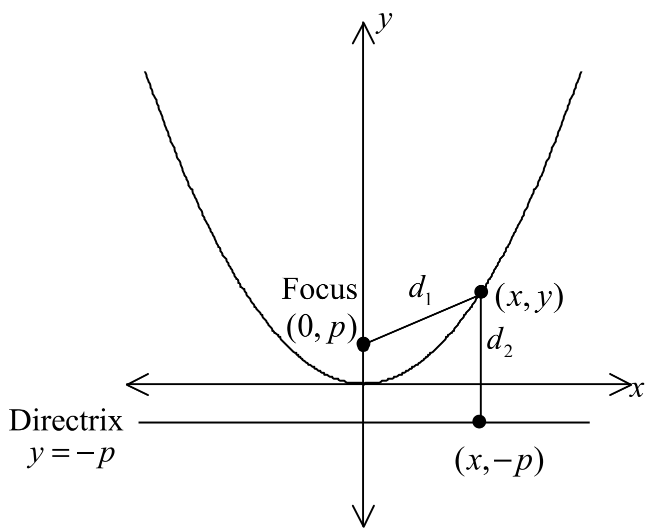
3.



4.



Derive the equation of a parabola from the definition:



Show $d_1 = d_2$.

$$\begin{aligned}
d_1 &= d_2 \\
\sqrt{(x-0)^2+(y-p)^2} &= \sqrt{(x-x)^2+(y-(-p))^2} && \text{Use distance formula} \\
\sqrt{x^2+(y-p)^2} &= \sqrt{(y+p)^2} && \text{Simplify} \\
x^2+(y-p)^2 &= (y+p)^2 && \text{Square both sides} \\
x^2+\cancel{y^2}-2py+\cancel{p^2} &= \cancel{y^2}+2py+\cancel{p^2} && \text{Simplify} \\
x^2-2py &= 2py \\
x^2 &= 4py && \text{Standard form (vertex at (0,0))}
\end{aligned}$$

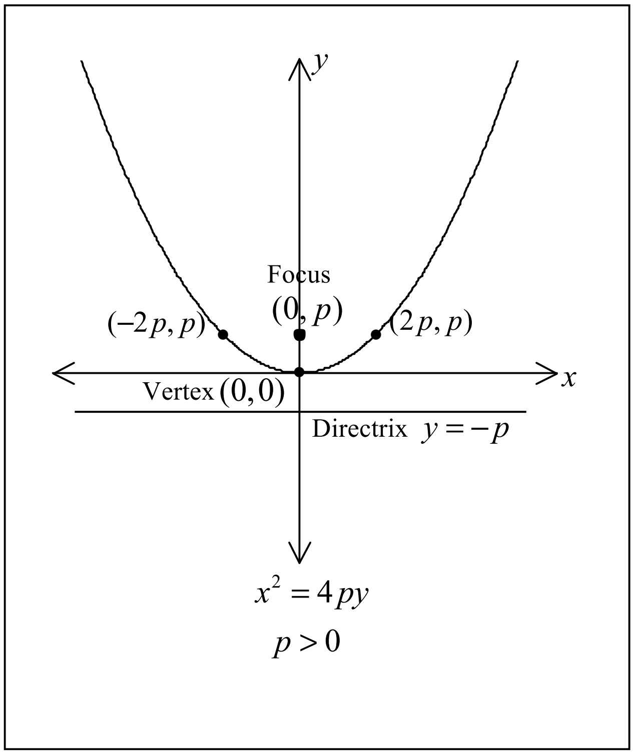
To graph a parabola given its equation in standard form, we can first locate the focus and vertex. Two other points can easily be located if we let $y = p$:

$$x^2 = 4py$$
$$x^2 = 4p^2$$
$$x = \pm 2p$$

Therefore, the following points will be on the graph:

x	y
$2p$	p
$-2p$	p

In your notes, draw and label a diagram based on the information above (work through this with the students step by step)



Ask: This drawing is for positive values of p. How would it look different if p is negative? [It will be concave down.]

Have students sketch and label a diagram for $p < 0$.

Now consider the inverse of $x^2 = 4py$:
How do we find the inverse of a relation? [Switch x and y]
So the inverse of $x^2 = 4py$ is $y^2 = 4px$.

How do the graphs of inverses compare? [They are reflections over the line $y = x$].
What will the graph of $y^2 = 4px$ look like? [The graph will open left or right.]

Graphing a parabola

Examples: Graph each parabola. Identify the focus and directrix of the parabola.

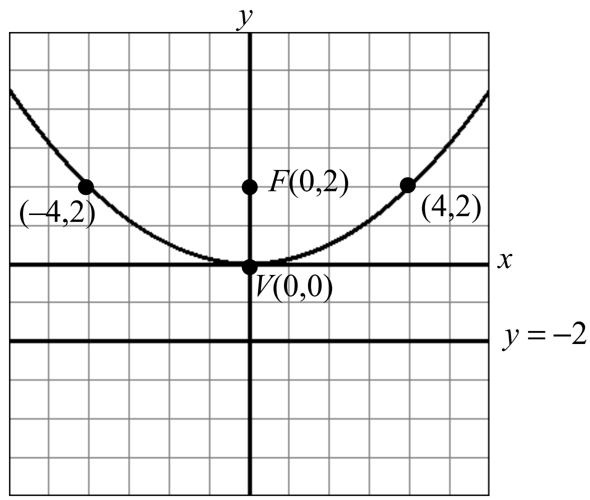
1) $x^2 = 8y$

This equation is in the form $x^2 = 4py$ with $p > 0$, so the parabola is concave up.

$4p = 8$

$p = 2$

Focus	Directrix
$(0, p)$	$y = -p$
$(0,2)$	$y = -2$



Let $y = p$

$y = 2$

Then

$x^2 = 4(2)^2$

$x^2 = 16$

$x = \pm 4$

x	y
4	2
–4	2

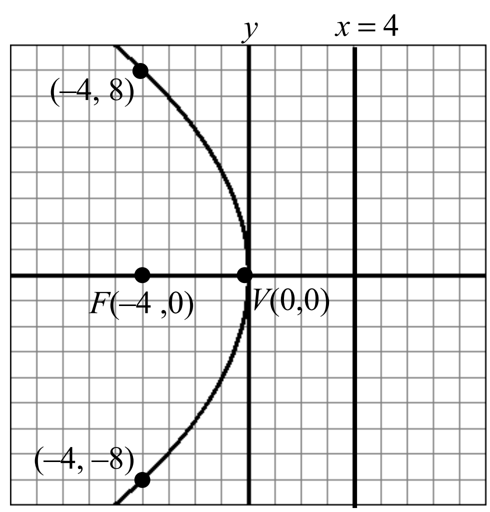
2) $y^2 = -16x$

This equation is in the form $y^2 = 4px$ with $p < 0$, so the parabola opens left.

$4p = -16$

$p = -4$

Focus	Directrix
$(p,0)$	$x = -p$
$(-4,0)$	$x = 4$



Let $x = p$

$x = -4$

Then

$y^2 = 4(-4)^2$

$y^2 = 64$

$y = \pm 8$

x	y
–4	8
–4	–8

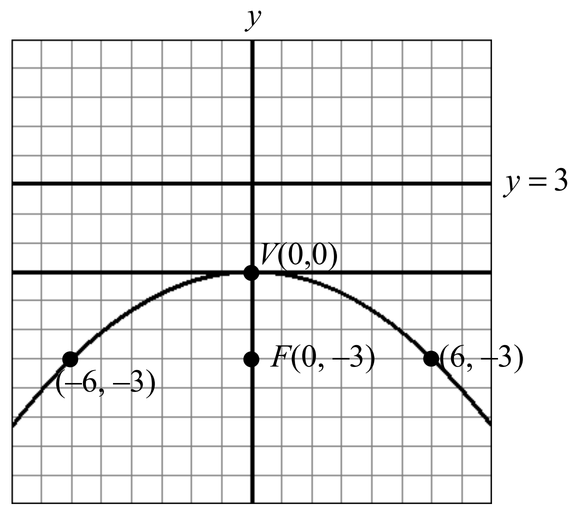
3) You Try: $x^2 = -12y$

This equation is in the form $x^2 = 4py$ with $p < 0$, so the parabola is concave down.

$4p = -12$
$p = -3$

Focus: $(0, p)$, $(0, -3)$

Directrix: $y = -p$, $y = 3$



Let $y = p$
$y = -3$
Then
$x^2 = 4(-3)^2$
$x^2 = 36$
$x = \pm 6$

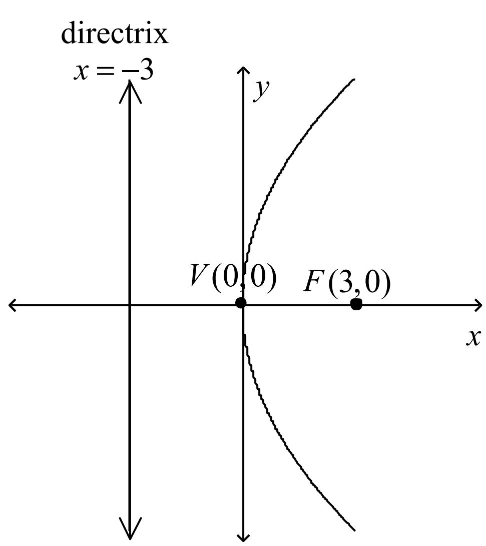
x	y
6	–3
–6	–3

Writing an equation for a parabola

Examples:

4) Write an equation in standard form for a parabola with vertex (0,0) and focus (3,0).

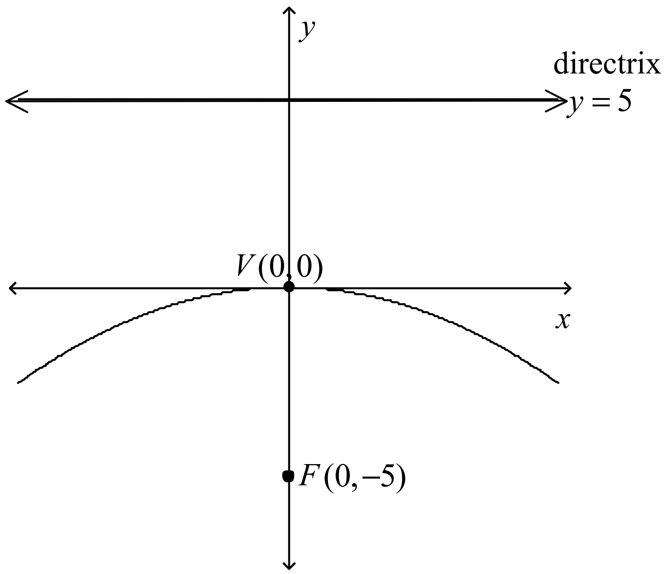
Start with a sketch:



The parabola opens to the right, so the standard form for the equation is $y^2 = 4px$. Also, since $p = 3$, the equation is:

$$y^2 = 4(3)x$$
$$y^2 = 12x$$

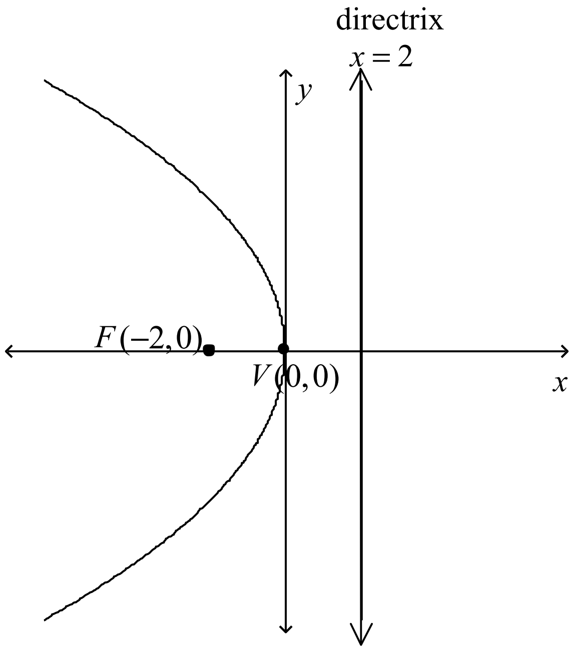
5) Write an equation in standard form for a parabola with vertex (0,0) and directrix $y = 5$
Start with a sketch:



The parabola is concave down, so the standard form for the equation is $x^2 = 4py$.
Also, since $p = -5$, the equation is:

$$x^2 = 4(-5)y$$
$$x^2 = -20y$$

6) YOU TRY: Write an equation in standard form for a parabola with vertex (0,0) and focus (–2, 0)
Start with a sketch:



The parabola opens to the left, so the standard form for the equation is $y^2 = 4px$.
Also, since $p = -2$, the equation is:

$$y^2 = 4(-2)x$$
$$y^2 = -8x$$